OPTIMAL COMPOSITION OF HYBRID/BLENDED REAL ESTATE PORTFOLIOS

Abstract

Purpose:

The purpose of this paper is to establish an optimum mix of liquid, publicly traded assets that may be added to a property fund portfolio to provide the enhanced liquidity required by institutional investors such as UK defined contribution pension funds which use these funds to gain exposure to real estate as an asset class. The performance and liquidity of various investments has become an issue of concern among investors especially as the ability of investors to exit these funds has frequently been compromised in periods following market shocks. Despite this focus on liquidity, few studies have analysed the structure and operation of open-ended real estate funds. For example, the amount of liquidity that these funds can offer and how they can structure their portfolios to meet the liquidity requirement of investors has received very little attention.

Design/Methodology/Approach:

The authors employ the mean-tracking error optimisation approach in determining the optimal combination of liquid assets that can be added to a property fund portfolio. The returns of the optimised portfolios are compared to the returns of the returns of existing property funds which employ the use of cash or listed real estate as a liquidity buffer. Multivariate Generalised Autoregressive models are used along with rolling correlations and tracking errors to gauge the effectiveness of the various portfolios in tracking the performance of the benchmark property index.

Findings:

The results indicate that applying formal optimisation techniques leads to a considerable improvement in the ability of the returns of blended real estate products to track the underlying benchmark property index even with significant addition of liquid, publicly traded assets.

Practical Implications:

The results imply that real estate fund managers can realise the liquidity benefits of incorporating publicly traded assets into their portfolios without sacrificing the ability to deliver property-like returns. However, a wider range of liquid assets must be considered, not just cash.

Originality/value:

To the authors' knowledge, this is the first study to analyse the optimal composition of liquid assets within blended or hybrid real estate portfolios.

Keywords: Real estate liquidity, tracking error, hybrid real estate, portfolio optimisation, blended real estate, defined contribution pensions

Article classification: Research paper

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Introduction

Unlisted real estate funds are an important part of many mature property markets around the world and have grown significantly in number and assets under management in the last two decades. Yet, despite this, there is still relatively little academic research on such funds, either on the structure and operation of the funds themselves or as an option for gaining exposure to real estate as an asset class. Open-ended funds, in particular, are a potentially attractive route for investors that desire exposure to a diversified pool of real estate investments while holding units that are reasonably liquid. However, the performance and liquidity of such funds has come into sharper focus in recent years. For instance, in the UK, the ability of investors to exit some open-ended funds has been restricted following market shocks (Forbes, 2012; 2017).

This makes questions around the degree of liquidity that such funds can offer, and the means by which they can do so, important issues for research. Open-ended fund units are not normally traded on a secondary market. Instead, units are normally bought or sold directly from the fund itself. In order to facilitate such trades, open-ended funds typically hold significant amounts of cash in the portfolio, for which appendix A provides evidence in respect of UK funds. In most market conditions, though, cash acts as a drag on fund performance, reducing the returns achieved and thus the ability of the fund to match the underlying real estate market (Frodsham, 2012). So, while holding more cash would enable a fund to redeem units more easily in downturns or following shocks, it also reduces its attractiveness to investors seeking real estate exposure.

In this context, this paper examines the implications of open-ended real estate funds holding different types of liquid assets in their portfolios alongside direct real estate. Such portfolios are called either blended or hybrid real estate portfolios, as they do not consist solely of direct real estate investments. Formal optimisation procedures are used to determine an optimal mix of liquid assets that might be held, with the aim of finding portfolios that replicate closely the performance of the underlying direct real estate market. The performance of these optimal portfolios is then compared to that of portfolios which use only a single predetermined liquid asset, such as cash, to provide the liquidity necessary for operation of the fund. The findings suggest that holding a mix of liquid assets could be more effective than holding cash in isolation.

This discussion does not imply that liquidity is a priority for all investors in real estate, not even for all investors in open-ended funds. For some investors in funds, restrictions on liquidity through minimum notice periods and exit fees are perceived to offer protection (Timmermans, 2009). Nonetheless, there has been increased emphasis on liquidity by numerous parties such as regulatory agencies, investment managers, pension trustees and consultants following the 2007-09 global financial crisis. At the same time, the low yield environment following the crisis has raised interest in real estate and alternative asset classes as a means of meeting

performance objectives. Thus, investors have been faced with the challenge of increasing their exposure to less liquid asset classes without sacrificing liquidity.

This paper contributes to the discussion of how real estate funds need to be structured to deal with the increased emphasis on liquidity while retaining the essential performance attributes of real estate as an asset class. The goal is to add liquid, tradable assets to a direct real estate portfolio but without altering the risk-return profile of the portfolio significantly. The paper begins by discussing literature on blended solutions in both real estate and other private asset markets before outlining the methods adopted to find optimal blended portfolios in a real estate context. The data used are then discussed before results and findings are presented, with the final section concluding on the implications of the findings and the areas for further research.

Literature review

Liquidity is a multi-faceted concept for which a variety of proxy measures exist, none of which capture all of its dimensions (Ametefe et al., 2016). Here, liquidity refers to the ability of investors to buy or sell assets quickly, at low cost and with minimal loss in value from executing the trade. Liquidity is also a relative concept, with direct real estate investments seen as comparatively illiquid owing to their high transaction costs, lengthy and uncertain trading times, and low frequency of transactions. Thus, in the absence of active secondary markets, real estate funds that want to offer greater liquidity to investors must do so by holding other assets in addition to direct real estate. This has been achieved traditionally through holding cash balances, but the use of public (or listed) real estate investments to facilitate greater liquidity has been explored recently by several studies.

Blending direct and listed real estate investments

Early studies into the benefits of including listed real estate in US direct real estate portfolios included Giliberto (1990), Giliberto and Testa (1990) and Stevenson (2001). These studies showed that there was potential to diversify by investing in both direct and listed real estate markets, with listed real estate assets acting as timing devices that enabled investors to observe market movements which take time to be reflected in direct real estate values. Stevenson (2001) conducted sector level analysis using three REIT sectors – equity, mortgage and hybrid REITs – as well as non-US listed real estate assets. He noted that, in addition to enhanced diversification, listed real estate also made it possible for an investor to quickly alter the exposure of their portfolio, as well as infuse the portfolio with liquidity as an alternative to cash.

NAREIT (2011) also examined the benefits of blending private and listed real estate investments. They found that optimal blends of private real estate funds and listed real estate assets produced significantly better risk-adjusted returns than investing in private vehicles alone. This was again driven by the diversification and timing benefits of listed real estate investments. They suggested that the optimal composition of blended real estate

portfolios should be around one-third listed real estate and two-thirds private real estate. The optimal blended real estate portfolio was found to produce positive annual returns, with not a single period of negative return over the entire sample period, which remarkably encompassed the 2007-09 global financial crisis.

Lee (2014) analysed portfolios containing a blend of private and public real estate using the 70:30 allocation suggested by NAREIT (2011). His study employed the percent contribution to risk measure of Holman and West (2013) to see whether the additional return generated by including listed real estate in the blended portfolio justifies the additional risk which it adds to the portfolio. The results showed that a blended public and private real estate portfolio produced a higher Sharpe ratio than any direct real estate fund type. Listed real estate was however found to be the main driver of volatility in the blended portfolios. Lee (2014) concluded that although listed real estate enhances the returns of real estate portfolios, the returns were not sufficient to justify the risk they contribute to the portfolio.

The findings of NAREIT (2011) were confirmed by Moss and Farrelly (2014). They analysed a 70:30 blend of UK unlisted real estate funds and global listed real estate funds over the period 1998-2013, as well as a portfolio split 75:25:5 between UK unlisted real estate, global listed real estate and cash. They found that adding global listed real estate to the portfolio resulted in return enhancement of about 19% over the full period. It also led to a significant increase in volatility, though they found that the Sharpe ratio only declined modestly due to the high increase in returns. They argued that this decline was acceptable given the additional liquidity benefits that were obtained by adding listed real estate. Meanwhile, the motivation for the cash allocation was to service the day-to-day liquidity requirements of the portfolio, so that the return enhancement benefits of listed real estate would not be lost through frequent trading of this element (see also Farrelly and Moss, 2014; Moss and Farrelly, 2015).

Nonetheless, Moss and Farrelly (2014) noted that, although many asset managers are aware of the benefits of including a proportion of listed real estate in their direct real estate portfolios, most were reluctant to implement this strategy. One of the main concerns was the increase in tracking error that would result from adding listed real estate to the direct real estate portfolio. For example, allocating 30% to listed real estate, as recommended by NAREIT (2011), resulted in a per-annum tracking error of 5.2% relative to the UK IPD direct real estate index. So, while listed real estate might offer a diversification benefit relative to direct real estate, if the aim in a multi-asset context is to obtain direct real estate returns, then adding listed real estate might be detrimental to that wider aim.

This raises the question of the extent to which listed and direct real estate could be considered as substitutes or complements from a multi-asset perspective, or even whether listed real estate should simply be considered part of the broader equity market. The earliest studies to examine these questions utilised simple correlation based tests, which often revealed a low contemporaneous correlation between direct and listed real estate, and

a high correlation between listed real estate and equities. More recent contributions have used cointegration and other advanced techniques to understand the linkages better.

Ling and Naranjo (1999) examined whether the direct real estate market and the REIT market in the US were integrated with the common equity market. While REITs were found to be integrated with the equity market, direct real estate markets were not. However, other studies have found evidence of integration between direct and listed real estate (for example, Wang et al., 1997; Tuluca et al., 2000; Morawski et al., 2008; Oikarinen et al., 2011). Most of these studies have found that returns in the listed real estate market lead direct real estate returns, implying that information is incorporated into the prices of listed real estate investments more quickly, and that the two types of real estate will not track each other closely in the short-term as a result.

Hoesli and Oikarinen (2012) examined the short-term and long-term dynamics between listed and direct real estate. Their analysis was based on sector level data from Australia, the UK and USA. The study also adjusted for the absence of leverage in direct real estate indexes. They show that over the long run, the returns of listed real estate were much closer to the direct real estate market than they are to the general stock market. Similarly, Yunus et al. (2012) found a long-term relationship between the listed and direct real estate markets, and that listed real estate leads the direct real estate market in the UK, US, Australia and the Netherlands. Meanwhile, Ang et al. (2013) studied the US market and found a common and highly persistent real estate cycle across both the direct and listed real estate markets. Both were broadly exposed to pro-cyclical market factors.

The foregoing suggests that direct and listed real estate might be good long-term substitutes, but, to the authors' knowledge, only Moss and Farrelly (2014; 2015) have considered the question of tracking error when combining direct and listed real estate into a blended portfolio, though low short-run correlations between direct and listed real estate imply that significant tracking error will be present. Meanwhile, none of the studies reviewed so far have addressed the question of which liquid assets beyond cash and listed real estate could be included within a blended real estate portfolio or the optimal combination of such assets. Given that many property funds have fixed, pre-determined allocation to various liquid assets, sometimes with a tolerance level, this study considers the optimal mix of liquid assets within the liquid asset component and examines the effects on returns, risk and tracking error in relation to a direct real estate benchmark.

Replicating returns of illiquid assets - evidence from other markets

The issue of enhancing liquidity within asset portfolios is not limited to the real estate market. O'Doherty et al. (2015) note a very high demand among institutional investors for passive replication products that track the performance of illiquid assets such as private equity and hedge funds. An example are Liquid Alternative Beta funds, which seek to replicate the risk and return characteristics of hedge fund indexes through investment in liquid, tradable instruments (see Drachman and Little, 2010). The use of factor models to replicate hedge fund performance with more liquid investments is perhaps the most popular approach among product developers

and academics (Hasanhodzic and Lo, 2007; Amenc, et al., 2008; Amenc et al., 2010; Bollen and Fisher, 2013). These models estimate the target fund or index exposure to certain factors and use the information to determine asset allocations within the replicated portfolios. For instance, Hasanhodzic and Lo (2007) constructed a factor model and used it to replicate the returns of 1,610 hedge funds. These funds covered all the major hedge fund investment strategies.

Although intuitively appealing, factor models have some drawbacks that limit their effectiveness in replicating hedge fund returns. The lack of transparency in the investment process of hedge funds makes it difficult to identify an appropriate set of factors. This leads to poor out-of-sample performance of these models in tracking hedge fund returns, while products based on them have also been found to underperform the target portfolios (Amenc et al., 2010; Bollen and Fisher, 2013). Kat and Palaro (2005) advocate an alternative approach that does not seek to generate identical period-to-period returns, but generate returns with the same statistical properties as the hedge fund being replicated. Meanwhile, other alternatives to factor models are the algorithmic approach and the payoff distribution approach. O'Doherty (2015) used an algorithm that combines information from several pre-selected models and use this to create a cloned hedge fund.

Private equity funds are perhaps the most illiquid alternative asset class, as capital in these funds can be locked up for as long as twelve years (Timmermans, 2009). Nonetheless, such funds appeal to institutional investors as they can offer higher returns and diversification opportunities. A few studies have examined the possibility of replicating the risk and return features of private equity funds using more liquid investments. Axelson et al. (2013) documented factors that determine the financial structure of private equity funds and compared these with publicly traded funds. Using the factors identified in Axelson et al. (2013), Stafford (2017) then explored the possibility of replicating private equity fund performance using a passive portfolio of similar public equity investments. A similar approach was taken by Ang et al. (2013), and MSCI has since created a liquid private equity index for US real estate investors based on their analysis.

Approach

Different strategies are employed in this study for the creation of blended or hybrid real estate portfolios. The aim is to construct blended real estate portfolios whose out-of-sample returns best replicate the risk and return features of the underlying direct real estate market over time. The types of blended portfolios constructed here are set out in Table I.

INSERT TABLE I AROUND HERE

The first strategy employs the use of cash as a liquidity buffer. This approach is referred to as Fund A and it is common among UK real estate funds as can be seen from Appendix A. The second strategy adds listed real estate to a portfolio of direct real estate investments. This is referred to as Fund B and is used in funds such as Legal & General's Pension Property Fund. Two further strategies are then examined using formal optimization procedures. In each case, portfolios are constructed by combining direct real estate with a wider selection of liquid assets; cash, listed real estate, aggregate stocks and bonds of various maturities. The third strategy, which is referred to as Fund C (DRE-ALL), does not have a minimum return requirement, while the fourth strategy, labelled Fund D (DRE-ALL1), includes a requirement that the returns of the portfolio equal at least the average total return on the IPD UK real estate index over the same period.

For the first and second portfolios, once the investment manager determines the proportion of cash or listed real estate that should be included in the blended real estate portfolio, the blending process consists of adding this proportion of liquid asset to the direct real estate portfolio. The return of the blended/hybrid portfolio can be obtained from the Equation 1 below:

$$R_p=[r_{DRE}*w_{DRE}]+[r_{LA}*w_{LA}] \quad (1)$$

Where:

R_p= Return of the blended real estate portfolio

r_{DRE} and r_{LA} = return of the direct real estate portfolio and selected liquid asset, respectively

w_{DRE} and w_{LA} = weight of direct real estate and the selected liquid asset in the blended portfolio

For the portfolios that include a wider selection of liquid assets, the optimal combination of such assets is determined as follows. First, an optimal allocation to the various liquid assets is determined using an extension of the mean-variance optimisation procedure of Markowitz (1952). The extension is made to accommodate the practice of evaluating the performance of managers relative to a benchmark (Rudd and Rosenberg, 1980; Roll, 1992; Rudolf et al., 1999). The optimisation problem is formulated in terms of tracking error and its volatility as opposed to absolute returns and its volatility. Tracking error is defined as the standard deviation of the difference between the portfolio returns and the benchmark return. In this context, it measures how closely the blended portfolio follows the returns on the benchmark index. Mathematically:

$$TE_P=\sigma(r_{Pt}-r_{bt}) \quad (2)$$

Another common approach to measuring the relationship between two variables is the correlation coefficient. However, tracking error is preferred here for the optimization procedure because the correlation coefficient is not a measure of congruence, but the strength of linear relationship. Thus, a high level of correlation is necessary but not a sufficient condition for minimising the tracking error variance of a portfolio. Given the variance of

the portfolio returns and benchmark returns as well as the correlation between a portfolio and the benchmark, the tracking error can be estimated using the relation below:

$$TE_P=\left(\sigma_P^2+\sigma_b^2-2\rho\sigma_p\sigma_b\right)^{0.5} \quad (3)$$

Where:
TE_P is the tracking error of a portfolio
σ_P^2 is the variance of portfolio returns
σ_b^2 the variance of benchmark returns and
ρ represents the correlation between the returns of the portfolio and the returns on the benchmark.

A general form of the tracking error optimisation model seeks to minimise the tracking error variance for a given expected excess return. The following numerical optimisation model is implemented:

$$\min_{w_k} \sum_{k=1}^{T}\left(r_{index,t}-\sum_{k=1}^{N} w_k r_{k,t}\right)^2 \quad (4)$$

Subject to:

$\sum_{t=1}^{T}\left(r_{index,t}-\sum_{k=1}^{N} w_k r_{k,t}\right)=0$

$\sum_{k=1}^{N} w_k=1$

$L<w_k<U$

Where:
$r_{index,t}$ = the return on the direct real estate benchmark at time t
$r_{k,t}$ = the return on the kth asset at time t
w_k = the weight assigned to the kth asset

The optimizer selects a combination of assets that provide the lowest tracking error relative to the IPD UK index returns, subject to the constraints of zero expected tracking error, unit sum of weights and a set allocation to direct real estate. The weight set for direct real estate ranged from 0% to 90%, in 10% intervals. The optimal combination of liquid assets was then determined for the remainder of the portfolio in each case. Discussion here, though, focuses on four particular allocations to the liquid asset component. A 10% weighting to liquid assets is discussed as this represents a typical allocation to liquid assets within UK open-ended real estate funds, while a 20% weight corresponds to the allocation by some existing hybrid funds such as the Legal & General Hybrid Real Estate fund. A 50:50 portfolio is also presented for comparison, while a pure replication portfolio made up entirely of liquid assets is included to demonstrate the possibility of using liquid assets alone to proxy the direct real estate market.

Tracking error optimisation models are subject to limitations. As they minimise in-sample tracking error with respect to a benchmark, this could lead to over-fitting the data in-sample at the expense of additional out-of-sample tracking error. The in-sample over-fitting may also result in an unstable portfolio structure that requires

frequent rebalancing and incurs significant transaction costs (Gregoriou et al., 2005). The models also make use of the covariance matrix, which means that they suffer from the weaknesses generated by the use of correlation as a measure of dependency. Correlation is a short-term statistic which lacks stability. Its estimation is sensitive to outliers, non-stationarity and volatility clustering. Some authors have suggested using vector autoregressive (VAR) models to measure the relationship between variables. For example, Alexander and Dimitriu (2004) compared the theoretical and empirical properties of the classic mean tracking error models with an enhanced MTE model that has an additional feature allowing for use of the cointegration between the tracking portfolio and the index. They found no clear advantages in using the enhanced version of the MTE model.

Nonetheless, it has been shown that tail events exist where parameters such as correlation change drastically. It is, therefore, better to calculate a conditional correlation which estimates correlation based on all information available up to a particular time point. Several approaches can be used to estimate the conditional correlation. A rolling correlation is easy to estimate and is capable of capturing time-variation and clustering of cross asset returns. However, there is no clear theoretical or empirical basis for selecting a window length. Furthermore, Anderson et al. (2008) observed that, since all the windows in a rolling correlation analysis are given the same weight, they tend to adjust very slowly to new information. This problem becomes greater with longer window lengths. There could be huge changes in correlation estimates when there are abnormally small or large return observations, especially when these observations enter or leave the window. Forbes and Rigobon (2002) found that rolling correlation coefficients tend to be prone to bias. They explained that, as volatility increases in one asset market, heteroscedasticity in returns may cause the correlation coefficient to be biased upward (see also Chong et al., 2012).

To make up for the drawbacks of the rolling correlation method, Engle (2002) suggests using the Dynamic Conditional Correlation (DCC) model. Many studies have used the DCC model within the real estate literature (Cotter and Stevenson, 2007; Chong et al., 2009; Liow et al., 2009; Fei et al, 2010; Case et al., 2012; Heaney and Sriananthakumar, 2012; Sing and Tan; 2013). The DCC model calculates the conditional correlations as a function of past volatilities of assets and the covariance between them. Given that all past information is used in the optimisation process, there is no difficulty in selecting a window length as with rolling correlations. Engle (2002) found that the multivariate and univariate volatility forecasts are consistent with each other. The volatility forecasts and the correlations of the original assets remain unchanged when new variables are added to the system, depending on the way the model is revised. Also, when applied to typical financial applications, it was found that DCC models revealed important time varying features that might otherwise be difficult to quantify.

The Dynamic Conditional Correlation model estimates a GARCH (1,1) specification, employing the resulting standardized residuals to estimate the time varying correlation matrix. In order to accomplish this, the residuals are transformed by their estimated standard deviations $\Xi_t = \varepsilon_t \Big/ \sqrt{h_t}$.

The covariance matrix can be expressed as $H_t \equiv D_t R_t D_t$, where D_t is a diagonal matrix of univariate GARCH volatilities. $R_t = Q_t^{*-1} Q_t Q_t^{*-1}$ is the time varying correlation matrix, with Q_t as described by:

$$Q_t = (1 - a - b)\overline{Q} + a\left(\Xi_{t-1}\Xi'_{t-1}\right) + Q_{t-1} \tag{6}$$

$\overline{Q}$ is the unconditional covariance of standardized residuals resulting from the first stage estimation, and Q_t^* is a diagonal matrix composed of the square root of the diagonal elements of Q_t. As with the standard GARCH (1, 1) model, the coefficients of the DCC(1, 1) model are estimated by maximum likelihood using the algorithm of Broyden–Fletcher–Goldfarb–Shanno (BFGS). The log-likelihood function, under the assumption of conditional multivariate normality can be displayed as follows:

$$L(\vartheta) = -\frac{1}{2}\left[TN \ln(2\pi) + \sum_{t=1}^{T} \ln\left|H_t + \Xi'_t H_t^{-1} \Xi_t\right|\right] \tag{7}$$

where Ξ_t is an N×1 vector stochastic process, with $H_t = E_{t-1}\left(\Xi_t \Xi'_t\right)$, being the N×N conditional variance-covariance matrix.

Data

This study uses UK data to analyse the effects of adding different combinations of liquid assets to a direct real estate portfolio over the period 1987 to 2015. Quarterly total return rates for direct real estate investments were sourced from MSCI IPD (splicing the IPD UK monthly index with the larger IPD UK quarterly index when latter begins in Q1 2000), while total returns for listed real estate, government bonds, equities and cash were all sourced from DataStream.[1] The in-sample portfolios were estimated from Q1 1987 as this was the earliest date from which quarterly return rates for direct real estate were available. Out-of-sample portfolios were estimated from the start of 1991, based on a five-year window. So the first out-of-sample portfolio is estimated using data from Q1 1987 to Q1 1991, then the next is estimated by rolling the window forward by one quarter, repeating this process up to the second quarter of 2015. In total, 78 out-of-sample portfolios are estimated.

One of the issues with direct real estate return series is that they are appraisal based, which means they tend to understate the risk of the underlying asset class. This leads, in turn, to an over-allocation to direct real estate in multi-asset optimisation studies (see Marcato and Key, 2007). In this study, the weight allocated to direct real estate has been set at particular thresholds and does not interfere with the relative allocation to different liquid

[1] Note that the opportunity set has been limited to UK assets to avoid the added complications of currency fluctuations. This omission does not detract from the general points that the study seeks to make.

assets in the portfolio. This is shown later by the results, as allocations within the liquid asset component remain stable, irrespective of the proportion allocated to direct real estate. However, the direct real estate index also serves as a benchmark against which the performance and tracking error of each portfolio is assessed, which is a potential limitation of the analysis conducted here.

Another criticism of direct real estate indexes is the difficulty in passively replicating their returns. The reason for this criticism is the belief that an investor must hold a large number of properties to diversify unsystematic risk. For example, while Callender et al. (2007) found that investing in 30 to 50 properties could achieve a large amount of risk reduction, they found that more properties were necessary to achieve very low levels of tracking error against the market index. However, their study was based on the use of naïve diversification, which ignores the potential gains from deliberate structuring of a portfolio to reduce systematic risk. Moreover, Boudry et al. (2013) have subsequently found that real estate portfolios do a good job of tracking index returns when these portfolios contain at least 20 assets.

The most common liquid assets in the portfolios of UK open-ended property funds are cash and listed real estate, but the liquid asset universe is expanded in this study to include the two main classes of liquid, publicly traded assets; bonds of various maturities and aggregate stocks. While many investors view the stable income flows of real estate to be bond-like, Shepard et al. (2015) found the long-run behavior of real estate returns to be more equity-like, i.e. cyclical and growth sensitive. Thus, the role that both bonds and general equities could play in replicating direct real estate returns is investigated. The UK FTSE all share index is used to represent the aggregate equity market while returns for bonds of various maturities are drawn from the Thomson Reuters DataStream bond indexes. The 3-month UK Treasury bill return is used as the proxy for cash.

Table II shows the quarterly return and risk characteristics of the various assets that are used to estimate optimal hybrid real estate portfolios. Listed real estate had the highest return and variability of all the selected liquid assets. Meanwhile, the returns for shorter-term bonds were found to be lower than those for longer-term bonds – implying an upward sloping yield curve over the majority of the period analysed.

INSERT TABLE II AROUND HERE

Concerning the relationship between direct real estate returns and the returns of the liquid assets, the correlation coefficient between direct real estate and listed real estate is the highest, followed by the correlation between direct real estate and stocks (Table 5(I)). It is interesting to note that the correlation between direct real estate and listed real estate is far greater than the correlation between the two stock series - listed real estate and general stocks. This may be indicative of the fact that listed real estate is more associated with the direct real estate market than the general stock market. Negative correlation between is observed between direct real estate all the bond maturities. However, the absolute value is higher for shorter maturity bonds than longer maturity

bonds. The correlation between the various bond maturities themselves is quite high, often more than 0.70. The correlation between cash and bonds is also higher for shorter maturity bonds than longer-maturity bonds.

Although the correlation coefficient between direct real state and listed real estate was the highest of all the selected liquid assets, due to the very high standard deviation, the tracking error with respect to direct real estate turned out to be very high. A high tracking error implies that, compared to the returns of returns the other liquid assets, the returns of listed real estate does not does not have a close association with the returns on the IPD direct real estate benchmark portfolio. This implies that adding general stocks or listed real estate to a blended real estate portfolio may result in the resulting portfolio exhibiting a risk and return profile that is quite different from those of the real estate benchmark. The tracking error between cash and direct real estate was the lowest, implying that cash may be the most suitable asset to be added to the direct real estate portfolio. The returns of shorter-term bonds exhibited lower tracking errors to direct real estate than longer-term bonds.

Results

The optimal allocations for the blended real estate portfolios are now discussed, looking first at the in-sample allocations and then the out-of-sample results. Funds A and B were made up of pre-determined allocations to direct real estate and either cash or listed real estate, with no optimization as per Moss and Farrelly (2014; 2015). Funds C and D were then constructed using the tracking error optimisation approach, with the aim of finding the combination of direct real estate and the selected liquid assets that produced the lowest tracking error. Fund C had no minimum return constraint and Fund D was constrained to produce returns that matched the average total returns on the direct real estate benchmark.

In-sample allocations, risk and returns of blended real estate portfolios

Prior to comparing risk and return for the various approaches, the allocations from the in-sample optimization exercise were as follows. Without minimum return constraints, Table III shows that about 80% of the allocation to liquid assets in Fund C went to cash, while the remaining allocation was to listed real estate and to general equities. Although the reported proportions change in line with the overall allocation to liquid assets, once the allocations are rescaled to reflect only the liquid component, the mix of liquid assets remains constant. The allocation to cash remains at 80%, while listed real estate and general stocks made up 12% and 8%, respectively. The high allocation that cash receives is broadly in line with the investments made by existing UK open-ended real estate funds. However, the addition of some listed real estate and general stocks could still improve the tracking error of these portfolios relative to the direct real estate market.

INSERT TABLE III AROUND HERE

For Fund D, with a constraint that the returns of the portfolio should at least equal the average return of direct real estate, long-term bonds with maturities greater than 10 years gain significant allocations (33% of the liquid asset component). Again, the proportion of each liquid asset to the total liquid asset allocation is very consistent. The allocation to cash dropped from 80% to about 25% of the liquid asset component, while the allocation to general stocks increased from 8% to about 24%, and listed real estate increased to 19%. This suggests that, to remove the negative performance impact of the so called cash drag, other types of liquid assets are likely to be required within a blended real estate portfolio.

An examination of the returns of the various blended real estate funds presented in Table III shows that the in-sample tracking error of Fund C (DRE-ALL) with respect to direct real estate was the lowest. This is followed by Fund A (DRE-CASH). However, the returns of the two portfolios that have the lowest tracking error were also quite low when compared to the direct real estate index. For example, with just a 10% allocation to liquid assets, there is a significant drop in return from 2.31% to 2.22% for Fund A and 2.25% for Fund C. In contrast, although Fund B (a mix of direct and listed real estate) produced the highest tracking error, it generated consistently higher returns than the direct real estate index, matching the findings of Farrelly and Moss (2014) and NAREIT (2011). The in-sample returns of Fund D, which includes the minimum return constraint, were also higher than the return on the direct real estate index, but with a far lower tracking error than the portfolios where listed real estate was used in isolation.

The foregoing discussion indicates that cash is the most suitable standalone asset to be included in direct real estate portfolios to improve liquidity without significantly altering the risk-return profile. However, adding listed real estate and a small amount of general stocks represents the optimal strategy. This optimal strategy still leads to a loss in returns because the unconstrained portfolios contain a large amount of cash, so the so-called cash drag remains. Thus, if concerned with return as well as tracking error, an investment manager may have to also include long-term bonds, especially those with maturities longer than ten years. More would have to be invested in listed real estate and general equities as well. The investor must accept a slightly higher tracking error if they require returns that are closer to direct real estate returns.

Out-of-sample allocations, risk and returns of blended real estate portfolios

As noted earlier, a shortcoming of the tracking error optimisation model is that the in-sample estimates may be over-fitted, which could result in higher out-of-sample tracking errors. How well the model performs depends on whether out-of-sample outcomes corroborate the in-sample results. Hence, in this section, the allocations and subsequent performance of blended real estate portfolios over different five year windows is studied. This analysis of the out-of-sample portfolios also enables us to see whether the composition of the optimal portfolios remains the stable across different windows or whether some rebalancing would be required.

As with the allocations obtained in-sample, the out-of-sample allocations for the various combinations of liquid assets remain similar, irrespective of the overall weight to liquid assets in the portfolio. This consistency implies that, once the optimal allocation is obtained, there is no need to re-run the allocation if the liquid asset weight is to be increased or decreased. All that is required is to rescale the allocations to individual liquid asset classes to reflect the new overall weight of liquid assets relative to direct real estate.

Figure 1 shows the liquid asset allocations within the blended real estate portfolios. The portfolios that have a 10%, 20% and 100% allocation to liquid assets are used as examples. It can be seen that the pattern of allocation remains the same irrespective of how much liquid asset is contained therein. The left hand panel of Figure 1 shows the results for the unconstrained optimal portfolios. The liquid component of these blended portfolios is invested heavily in cash, especially prior to 2007. The only assets that had a significant allocation in the liquid component apart from cash is listed real estate. After 2007, though, general stocks and listed real estate together received allocations averaging about 40% of the liquid asset allocations.

INSERT FIGURE 1 AROUND HERE

The right-hand panel of Figure 1 shows the allocation in the constrained optimal portfolios. Clearly, these portfolios are more diversified than those in the left panel. The four assets that received significant allocations here are cash, listed real estate, general stocks and long-term bonds. Long-term bonds (10+ years) dominate the allocations between 1997 and 2003. Cash dominated the portfolio prior to 2000 and after 2007 but does not gain any allocations between 2000 and 2007. Short and medium term (3, 5 and 7 year) bonds also received allocations at various points within these portfolios.

Figure 2 shows the returns of the various blended real estate series alongside the returns on the IPD UK index. It can be seen that the returns of the blended Fund A (containing cash) and Fund C (unconstrained portfolio drawing on all liquid assets) track the benchmark more closely than their counterparts. This is also apparent from Table IV, where the tracking error reported for Fund C (DRE-ALL) is the lowest, followed by that of Fund A (DRE-CASH). The combination of listed and direct real estate (Fund B) had the highest tracking error. This result was consistent across all levels of liquid asset allocation. The tracking error per quarter for Fund C ranges from 0.29% for the portfolio with only 10% weighting to liquid assets to 2.90% for that which contains only liquid, publicly traded assets. For Fund A, the tracking error ranges from 0.32% for the portfolio with 10% liquid assets to 3.16% where cash is the only asset in the portfolio.

INSERT FIGURE 2 AROUND HERE

INSERT TABLE IV AROUND HERE

With only 10% allocated to listed real estate, the tracking error for Fund B is 1.06%. This increases to 10.56% tracking error per quarter where only listed real estate is held. Imposing a minimum return requirement on the

minimum tracking error model also increases the tracking error, but not as much as observed for the listed real estate and direct real estate mix. The tracking error ranged from 2.64% to 8.82%.

Table IV shows that of the different blended portfolios, the only one that recorded return enhancement when the allocation to liquid assets was increased was Fund B. However, there are differences when compound growth is considered instead of arithmetic average return rate. This shows that using listed real estate alone only enhanced returns up to a certain threshold. This was due largely to the volatility in listed real estate returns. Listed real estate also has the highest drawdown – a measure of risk which indicates how much an investment value would fall from peak-to-through until a new maximum is reached. Consequently, the portfolios for Fund B recorded the highest standard deviations among all the portfolios constructed.

None of the other portfolios showed an increase in return with the addition of liquid assets. On the contrary, the returns for Fund A (which contains cash and listed real estate) falls with every increase in the liquid asset allocation. Table VI shows that as much as 53% can be lost by substituting direct real estate for cash. This means that the lower tracking error observed earlier for the blended portfolio containing cash often comes at the cost of significant losses in return. The challenge then is to find a way of reducing tracking error without sacrificing significant returns.

Including a wider selection of liquid assets and employing the Minimum Tracking Error optimisation procedure results in lower tracking error than simply adding cash to a direct real estate portfolio. From Table IV, it can be seen that Fund C (DRE-ALL) produced lower tracking errors than Fund A (DRE-CASH), while providing returns that were higher. The loss in return required to achieve this low tracking error was 37%, compared to the 53% observed for Fund A. Meanwhile, in the case of Fund D (DRE-ALL1), unlike the in-sample results – which were subject to the minimum return constraint – the out-of-sample returns fell short of the returns on the IPD benchmark. This notwithstanding, the returns obtained from Fund D were higher and closer to direct real estate returns than those obtained for Fund C (DRE-ALL). Yet the tracking error increased slightly with the imposition of the minimum return constraint.

Analysis of the time varying relationship between hybrid fund returns and direct real estate

To account for potential temporal instability in relationships, rolling tracking errors are estimated using a 20-quarter (five year) window. As can be seen from Figure 3, the Portfolio containing direct real estate and listed real estate consistently had the highest tracking error relative to the direct real estate benchmark over the whole period. This confirms the results of previous studies and concerns raised by industry practitioners regarding the incorporation of listed real estate in direct real estate portfolios and also the results of the static tracking error presented earlier. The best combination remains cash and the blended real estate Fund C – which contains all liquid assets.

INSERT FIGURE 3 AROUND HERE

Tracking error generally increased for all the hybrid real estate funds during the periods around the recent Global Financial Crisis (2007 – 2009). Appendix 5(C) contains the summary statistics for the 20 quarter rolling tracking error for the four blended real estate funds. As indicated earlier, the blended real estate fund C (containing all liquid assets) recorded the lowest tracking errors which ranged from a minimum of 0.0162% per quarter (containing a 10% allocation to liquid assets) to a maximum of 5.4733 per quarter (for the pure replication fund). This means that even with no allocation to direct real estate, the maximum tracking error recorded for blended real estate fund C was less than 6% per quarter. This compares to a minimum of 0.0695% to a maximum of 22.49% tracking error for the blended real estate fund B (listed real estate – direct real estate mix). Blended real estate Funds A and D recorded a range of 0.0256% – 8.29% and 0.0256% - 6.92% respectively.

Different estimates of correlation are presented in Table V. The average dynamic conditional correlation was identical to the 20 quarter rolling correlation for most funds. As observed by Chung et al. (2012) and Forbes and Rigobon (2002), where there were differences, the average conditional correlation was mostly lower than the average rolling correlation. At lower levels of direct real estate allocation, the correlation between Fund A (DRE-CASH) was very low, even negative at times. However, with very little allocation to direct real estate, the correlation between hybrid real estate Fund A and direct real estate increases remarkably. Fund A mostly had a higher correlation to direct real estate than all the other funds. This is attributable to the very low volatility of cash which makes it less likely to significantly alter the return pattern of the direct real estate portfolio when added to this portfolio.

INSERT TABLE V AROUND HERE

Interestingly, blended real estate fund B (LRE-DRE) recorded very stable correlation pattern relative to direct real estate. Even for the pure replication portfolio, containing no allocation to direct real estate, the correlations coefficients were above 0.45 over the entire sample period. This is due to the fact that as a stand-alone asset, listed real estate had a very high correlation with direct real estate. Consequently, when correlation is used as the main measure of association, blended portfolios containing listed real estate show a stable correlation pattern with direct real estate. What this result implies is that for investors who wish to have real estate portfolios that have significant allocations to liquid assets, especially more than 50%, listed real estate may represent a good choice of asset than any other stand-alone asset. However, it is clear from the foregoing discussion that a multi-asset approach is best at producing the best blended or hybrid real estate portfolios.

Conclusion

Managers of open-ended real estate funds have typically used cash, and sometimes listed real estate, to enhance the liquidity profile of their portfolios. Focus on the performance and liquidity of such funds has increased in the wake of market shocks and with the increased emphasis on liquidity by institutional investors, including the rising number of Defined Contribution pension funds. DC pension funds have been offering daily traded funds to contributors as opposed to monthly or even quarterly traded funds, raising the question as to whether real estate is a possible option in such a framework. Thus, with these developments, there is a need to design real estate funds with adequate liquidity to meet the requirements of such investors.

The inclusion of listed real estate in direct real estate portfolios has been found to enhance the returns of such portfolios, along with providing more liquidity. However, many investment managers are reluctant to use listed real estate within blended real estate portfolios owing to the fact that its inclusion can result in a high tracking error relative to a direct real estate benchmark – implying that the resulting portfolio fails to provide the investor with property-like returns. Meanwhile, the use of cash has been found to result in significant drags on portfolio return. The challenge for real estate funds then is to find a way of minimising tracking error with the direct real estate market without significant loss of returns or alteration to the fundamental performance features of direct real estate assets.

This paper explores the possibility of expanding the asset universe beyond cash and listed real estate to see if it is possible to produce portfolios that deliver property-like returns along with enhanced liquidity. In addition to cash and listed real estate, general stocks and bonds of various maturities were used as options for addition to direct real estate portfolios. To create the blended real estate portfolios, Minimum Tracking Error optimisation procedures were utilised. This procedure is an extension of the classic Mean-Variance optimisation procedure and was implemented with and without a minimum return constraint in order to observe the most effective combinations of liquid assets in meeting portfolio objectives.

The results show that using a wider array of assets produced lower tracking errors than those obtained by using a cash-only liquidity buffer. The returns obtained were higher than those obtained through a direct real estate and cash mix, without significant increase in tracking error. In comparison, the returns of the direct real estate and listed real estate combination did not perform well in replicating the performance of the underlying direct real estate market. As in other studies, the direct-listed real estate combination produced enhanced returns, but this study shows that the terminal value obtained over the period for this strategy was lower if a certain threshold allocation to listed real estate was crossed. This is due to the high volatility and drawdown of listed real estate as a stand-alone asset.

One key question which this paper seeks to answer is which assets should be included in the blended/hybrid real estate portfolio. The results from this study shows that the answer depends on whether the fund manager is concerned solely with tracking direct real estate returns as closely as possible or is also concerned with earning

returns that are not significantly lower than direct real estate returns. A pure, unconstrained tracking error minimization portfolio consists largely of cash along with a limited amount of listed real estate and general stocks. The dominance of cash in this portfolio lends some credence to the current allocation within UK unlisted real estate fund portfolios. However, imposing a minimum return constraint where the portfolio must at least match the average return of the direct market resulted in a more diversified portfolio, with cash playing a limited role. These constrained portfolios had significant allocations to long term bonds, listed real estate and general stocks.

A number of future studies could be conducted in this area. The asset universe could be expanded further to include non-UK real estate and liquid assets. This would result in additional challenges stemming from foreign exchange risk and the difficulty in finding foreign assets that can be incorporated without significantly changing the risk and return profile of the underlying direct real estate portfolio. The use of factor models, as done in hedge fund and private equity markets, could also be explored to determine how direct real estate returns might be replicated to enable investors take advantage of the benefits of direct real estate investments with minimal liquidity risk. Finally, this study used historical returns in the estimation of the portfolio weights. Hence, another area that could be explored is the use of forward looking risk and return measures in the construction of the blended real estate portfolios.

References

Alexander, C. & Dimitriu, A. (2004), *The art of investing in hedge funds: Fund selection and optimal allocations*. Retrieved from: https://papers.ssrn.com/sol3/papers.cfm?abstract_id=494404 [Accessed March 2018]

Amenc, N., Géhin, W., Martellini, L. & Meyfredi, J. C. (2008), "Passive hedge fund replication: A critical assessment of existing techniques", *Journal of Alternative Investments*, Vol. 11, No. 2, pp. 69-83.

Amenc, N., Martellini, L., Meyfredi, J. C. & Ziemann, V. (2010), "Passive hedge fund replication – Beyond the linear case", *European Financial Management*, Vol. 16, No. 2, pp. 191-210.

Ametefe, F., Devaney, S. & Marcato, G. (2016), "Liquidity: A Review of Dimensions, Causes, Measures, and Empirical Applications in Real Estate Markets", *Journal of Real Estate Literature*, Vol. 24, No. 1, pp. 1-29.

Anderson, R. M. & Raimondo, R. C. (2008), "Equilibrium in Continuous-Time Financial Markets: Endogenously Dynamically Complete Markets", *Econometrica*, Vol. 76, No. 4, pp. 841-907.

Ang, A., Nabar, N. & Wald, S. J. (2013), "Searching for a common factor in public and private real estate returns", *Journal of Portfolio Management*, Vol. 39, No. 5, 120-133.

Axelson, U. & Martinovic, M. (2013), *European venture capital: myths and facts*, London: British Private Equity & Venture Capital Association.

Bollen N. & Fisher G. (2013), "Send in the Clones? Hedge Fund Replication with Futures Contracts", *Journal of Alternative Investments*, Vol. 16, No. 2, pp. 80-95.

Boudry, W. I., Coulson, N. E., Kallberg, J. G. & Liu, C. H. (2013), "On Indexing Commercial Real Estate Properties and Portfolios", *Journal of Real Estate Finance and Economics*, Vol. 47, No. 4, pp. 617-639.

Callender, M., Devaney, S., Sheahan, A. & Key, T. (2007), "Risk reduction and diversification in UK commercial property portfolios", *Journal of Property Research*, Vol. 24, No. 4, pp. 355-375.

Case, B., Yang, Y. & Yildirim, Y. (2012), "Dynamic correlations among asset classes: REIT and stock returns", *Journal of Real Estate Finance and Economics*, Vol. 44, No. 3, pp. 298-318.

Chong, J., Miffre, J. & Stevenson, S. (2009), "Conditional correlations and real estate investment trusts", *Journal of Real Estate Portfolio Management*, Vol. 15, No. 2, pp. 173-184.

Chong, J., Krystalogianni, A. & Stevenson, S. (2012), "Dynamic correlations between REIT sub-sectors and the implications for diversification", *Applied Financial Economics*, Vol. 22, No. 13, pp. 1089-1109.

Cotter, J. & Stevenson, S. (2007), "Uncovering volatility dynamics in daily REIT returns", *Journal of Real Estate Portfolio Management*, Vol. 13, No. 2, pp. 119-128.

Drachman, J. & Little, P. (2010) *Enhancing Liquidity in Alternative Portfolios*, Credit Suisse: www.credit-suisse.com/pwp/am/downloads/marketing/tl_201006_lab_us_apac_jp__ie_lux_sp_scand_uk_.pdf
[Accessed March 2018]

Engle, R. (2002), "Dynamic conditional correlation: A simple class of multivariate generalized autoregressive conditional heteroskedasticity models", *Journal of Business & Economic Statistics*, Vol. 20, No. 3, pp. 339-350.

Farrelly, K. & Moss, A. (2014). "Blending Public and Private Real Estate Allocations for Defined Contribution Pension Funds: A UK Case Study", *Journal of Real Estate Portfolio Management*, Vol. 20, No. 3, pp. 137-150.

Fei, P., Ding, L. & Deng, Y. (2010), "Correlation and volatility dynamics in REIT returns: performance and portfolio considerations", *Journal of Portfolio Management*, Vol. 36, No. 2, 113-125.

Forbes, J. (2012), *Unlisted funds – Lessons from the crisis*, Report for The Association of Real Estate Funds, PricewaterhouseCoopers LLP.

Forbes, J. (2017), *A review of real estate fund behaviour following the EU referendum*, Report for The Association of Real Estate Funds. John Forbes Consulting LLP.

Forbes, K. J. & Rigobon, R. (2002), "No contagion, only interdependence: measuring stock market comovements", *The Journal of Finance*, Vol. 57, No. 5, pp. 2223-2261.

Frodsham, M. (2012), *A decade of fund returns*, London: Investment Property Forum.

Giliberto, M. (1990), *Global Real Estate Securities: Index Performance and Diversified Portfolios*, Salomon Brothers Bond Market Research, February, pp. 1-9.

Giliberto, M. & Testa, B. (1990), *Global Property Share Performance by Geographic Region*, Salomon Brothers Bond Market Research, August, pp. 1-6.

Gregoriou, G. N., Sedzro, K. & Zhu, J. (2005), "Hedge fund performance appraisal using data envelopment analysis", *European Journal of Operational Research*, Vol. 164, No. 2, pp. 555-571.

Hasanhodzic, J. & Lo, A. W. (2007), "Can hedge-fund returns be replicated? The linear case", *Journal of Investment Management*, Vol. 5, No. 2, pp. 5-45.

Heaney, R. & Srianantha kumar, S. (2012), "Time-varying correlation between stock market returns and real estate returns", *Journal of Empirical Finance*, Vol. 19, No. 4, pp. 583-594.

Hoesli, M. & Oikarinen, E. (2012), "Are REITs real estate? Evidence from international sector level data", *Journal of International Money and Finance*, Vol. 31, No. 7, pp. 1823-1850.

Holman, J. and West, K. (2013), "A 'Required Return' Framework is a Useful Tool for Managing Diversified Hedge Fund Portfolios", *Hedge Funds Review*, 29 May.

Kat, H. M. & Palaro, H. P. (2005), *Who needs hedge funds? A copula-based approach to hedge fund return replication*. Retrieved from: https://papers.ssrn.com/sol3/papers.cfm?abstract-id_855424 [Accessed March 2018]

Lee, S. (2014), "The Contribution Risk of REITs in the Blended Public and Private Real Estate Portfolio", *Real Estate Finance*, Vol. 30, No. 5, pp. 50-55.

Ling, D. C. & Naranjo, A. (1999), "The integration of commercial real estate markets and stock markets", *Real Estate Economics*, Vol. 27, No. 3, pp. 483-515.

Liow, K. H., Ho, K. H. D., Ibrahim, M. F. & Chen, Z. (2009), "Correlation and volatility dynamics in international real estate securities markets", *Journal of Real Estate Finance and Economics*, Vol. 39, No. 2, pp. 202-223.

Marcato, G. & Key, T. (2007), "Smoothing and Implications for Asset Allocation Choices", *Journal of Portfolio Management*, Vol. 33, No. 5, pp. 85-98.

Markowitz, H. (1952), "Portfolio Selection", *Journal of Finance*, Vol. 7, No. 1, pp. 77-91.

Morawski, J., Rehkugler, H. & Fuss, R. (2008), "The nature of listed real estate companies: property or equity market?", *Financial Markets and Portfolio Management*, Vol. 22, No. 2, pp. 101-126.

Moss, A. & Farrelly, K. (2014), *The performance implications of adding global listed real estate to an unlisted real estate portfolio: A case study for UK Defined Contribution funds*. European Public Real Estate Association.

Moss, A. & Farrelly, K. (2015), "The performance of a blended real estate portfolio for UK DC investors", *Journal of Property Investment & Finance*, Vol. 33, No. 2, pp. 156-168.

NAREIT (2011), *Optimizing Risk and Return in Pension Fund Real Estate: REITs, Private Equity Real Estate and the Blended Portfolio Advantage*, National Association of Real Estate Investment Trusts. Retrieved from: www.reit.com/sites/default/files/portals/0/PDF/PortfolioOptimizer.pdf [Accessed March 2018]

O'Doherty, M. S., Savin, N. E. & Tiwari, A. (2015), "Evaluating hedge funds with pooled benchmarks", *Management Science*, Vol. 62, No. 1, pp. 69-89.

Oikarinen, E., Hoesli, M. & Serrano, C. (2011), "The Long-Run Dynamics between Direct and Securitized Real Estate", *Journal of Real Estate Research*, Vol. 33, No. 1, pp. 73-102.

Pagliari, J. L., Scherer, K. A. & Monopoli, R. T. (2005), "Public Versus Private Real Estate Equities: A More Refined, Long-Term Comparison", *Real Estate Economics*, Vol. 33, No. 1, pp. 147-187.

Roll, R. (1992), "A Mean/Variance Analysis of Tracking Error", *Journal of Portfolio Management*, Vol. 18, No. 4, 1992, pp. 13-22.

Rudd, A. & Rosenberg, B. (1980), The "Market Model" in Investment Management. *The Journal of Finance*, Vol. 35, No. 2, pp. 597-607.

Rudolf, M., Wolter, H. J. & Zimmermann, H. (1999), "A linear model for tracking error minimization", *Journal of Banking & Finance*, Vol. 23, No. 1, pp. 85-103.

Shepard, P., Hobbs, P. & Liu, Y (2015) *Is Real Estate Bond-Like? Growth Drives Long-Run Risk and Return*, MSCI. Retrieved from: www.msci.com/www/research-paper/is-real-estate-bond-like-/0182632204 [Accessed March 2018]

Sing, T. F. & Tan, Z. Y. (2013), "Time-varying correlations between stock and direct real estate returns", *Journal of Property Investment & Finance*, Vol. 31, No. 2, pp. 179-195.

Stafford, E. (2017), *Replicating Private Equity with Value Investing, Homemade Leverage, and Hold-to-Maturity Accounting.* Retrieved from: https://ssrn.com/abstract=2720479 [Accessed March 2018]

Stevenson, S. (2001), "The Long-Term Advantages to Incorporating Indirect Securities in Direct Real Estate Portfolios", *Journal of Real Estate Portfolio Management*, Vol. 7, No. 1, pp. 5-16.

Timmermans, X. (2009), "Investing in illiquid assets", *Society of Actuaries: Risks and Rewards*, Vol. 53, pp. 12-16.

Tuluca, S., Myer, F. & Webb, J. (2000), "Dynamics of Private and Public Real Estate Markets", *Journal of Real Estate Finance and Economics*, Vol. 21, No. 3, pp. 279-296.

Wang, P., Lizieri, C. & Matysiak, G. (1997), "Information asymmetry, long-run relationship and price discovery in property investment markets", *The European Journal of Finance*, Vol. 3, No. 3, pp. 261-275.

Yunus, N., Hansz, J. & Kennedy, P. (2012), "Dynamic Interactions between Private and Public Real Estate Markets: Some International Evidence", *Journal of Real Estate Finance and Economics*, Vol. 45, No. 4, pp. 1021–1040.

Table I: Composition of Various Blended Real Estate Portfolios

Fund A (DRE-CASH)	Direct real estate and cash
Fund B (DRE-LRE)	Direct real estate and listed real estate
Fund C (DRE-ALL)	Direct real estate, cash, listed real estate, aggregate stocks, bonds of various maturities (No minimum return constraint)
Fund D (DRE-ALL1)	Direct real estate, cash, listed real estate, aggregate stocks, bonds of various maturities (With minimum return constraint)

Note: The minimum return constraint is that the target return should be equal to or greater than the average return on the IPD All Property Index. See text for further discussion.

Table II: Summary Statistics and Correlation Coefficients for Quarterly Total Return Rates (Q1 1987 – Q1 2015)

	Direct Real Estate	**Listed real estate**	**General stocks**	**Bonds 10yr+**	**Bonds 10yr**	**Bonds 7yr**	**Bonds 5yr**	**Bonds 3yr**	**Bonds 2yr**	**Cash**
Panel A: summary statistics										
Mean	2.31	2.90	2.70	2.40	2.24	2.06	1.89	1.75	1.60	1.36
Median	2.59	4.72	3.77	2.14	2.34	1.99	1.73	1.64	1.30	1.31
Maximum	9.92	43.98	27.11	15.96	15.12	9.45	10.66	7.32	5.92	3.48
Minimum	-12.96	-34.18	-30.61	-9.38	-8.26	-5.91	-4.77	-2.12	-1.12	0.09
Std. Dev.	3.06	12.03	8.44	4.67	3.99	3.11	2.64	1.81	1.45	0.90
Sharpe ratio	0.31	0.13	0.16	0.22	0.22	0.22	0.20	0.21	0.16	0.00
TE w.r.t. DRE	0.00	11.00	8.27	5.76	5.41	4.83	4.46	3.94	3.74	3.26
Panel B: Correlation Coefficients										
Direct real estate	1.0000									
Listed real estate	0.4514	1.0000								
General stocks	0.2388	0.0637	1.0000							
Bonds 10yr +	-0.0704	0.0848	0.0662	1.0000						
Bonds 10yr	-0.1637	0.0501	0.0751	0.9613	1.0000					
Bonds 7yr	-0.2235	0.0536	0.0792	0.9082	0.9757	1.0000				
Bonds 5yr	-0.2218	0.0472	0.0816	0.8542	0.9475	0.9765	1.0000			
Bonds 3yr	-0.2624	0.0159	0.1136	0.7498	0.8604	0.9163	0.9591	1.0000		
Bonds 2yr	-0.2850	-0.0279	0.0846	0.6318	0.7556	0.8220	0.8850	0.9668	1.0000	
Cash	-0.0787	-0.0675	0.0288	0.0404	0.1057	0.1499	0.2248	0.3584	0.5140	1.0000

Note: Bonds 10yr+ = Bonds with maturity greater than 10 years; Bonds 10yr = 10 year bonds; Bonds 7yr = 7 year bonds; Bonds 5yr = 5 year bonds; Bonds 3yr = 3 year bonds; Bonds 2yr = 2 year bonds; TE w.r.t. DRE = Tracking error with respect to IPD All Property index; Sharpe ratio = Risk-adjusted returns with respect to the risk-free rate i.e. 3-month UK T-bill rate; Std. Dev. = Standard deviation of returns.

Table III: In-Sample Statistics of Blended/Hybrid Real Estate Portfolios

PERFORMANCE STATISTICS					ALLOCATIONS									
Portfolio Return	Excess return wrt DRE	Portfolio Standard Deviation	Tracking Error	Correlation with Real Estate	Stocks	Bonds (10+ years)	Bonds (10 year)	Bonds (7 years)	Bonds (5 years)	Bonds (3 years)	Bonds (2 years)	Listed real estate	Cash	Direct real estate
Fund A Direct real estate and cash														
1.3624	-0.9505	0.9028	3.3561	-0.0787	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00
2.0278	-0.2851	2.1387	1.0068	0.9920	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.30	0.70
2.1228	-0.1901	2.4415	0.6712	0.9973	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.20	0.80
2.2178	-0.0950	2.7495	0.3356	0.9946	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.10	0.90
Fund B Direct real estate and listed real estate														
2.9025	0.5897	12.0337	11.0032	0.4433	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00
2.4976	0.1847	5.0047	3.3010	0.7627	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.30	0.00	0.70
2.4397	0.1268	4.1977	2.2006	0.8578	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.20	0.00	0.80
2.3819	0.0690	3.5517	1.1003	0.9527	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.10	0.00	0.90
Fund C Direct real estate and all liquid assets (No minimum return constraint)														
1.6611	-0.6287	1.7745	2.7989	0.4332	0.08	0.00	0.00	0.00	0.00	0.00	0.00	0.12	0.80	0.00
2.1174	-0.1886	2.4215	0.8397	0.9802	0.03	0.00	0.00	0.00	0.00	0.00	0.00	0.04	0.24	0.70
2.1872	-0.1257	2.6316	0.5598	0.9925	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.02	0.16	0.80
2.2477	-0.0629	2.8365	0.2799	0.9984	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.08	0.90
Fund D Direct real estate and all liquid assets (with minimum return constraint)														
2.3129	0.0000	3.6384	3.7454	0.3828	0.23	0.33	0.00	0.00	0.00	0.00	0.00	0.19	0.25	0.00
2.3162	0.0033	2.7587	1.1236	0.9293	0.07	0.10	0.00	0.00	0.00	0.00	0.00	0.06	0.07	0.70
2.3151	0.0022	2.8128	0.7491	0.9703	0.05	0.07	0.00	0.00	0.00	0.00	0.00	0.04	0.05	0.80
2.3140	0.0011	2.9153	0.3745	0.9931	0.02	0.03	0.00	0.00	0.00	0.00	0.00	0.02	0.02	0.90

Table IV: Quarterly Out-Of-Sample Summary Statistics of Blended Real Estate Portfolios

Liquid asset percentage	Selection of liquid assets				
10 percent liquid (90% direct real estate)	**FUND A (DRE_CASH)**	**FUND B (DRE_LRE)**	**FUND C (DRE_ALL)**	**FUND D (DRE_ALL1)**	**DRE**
Tracking error with respect to DRE (%)	0.3157	1.0561	0.2909	0.4037	0.0000
Average return (%)	2.0885	2.2462	2.1249	2.1882	2.2065
Average excess returns (%)	-0.1180	0.0397	-0.0816	-0.0183	0.0000
Standard deviation (%)	2.7127	3.4542	2.8128	2.9565	3.0216
Sharpe ratio	0.3825	0.3461	0.3819	0.3847	0.3825
Index value	670.05	773.36	692.37	731.03	742.18
Maximum drawdown	0.3145	0.4003	0.4110	0.3877	0.3886
Change in return with respect to DRE (%)	-5.3471	1.8013	-3.6981	-0.8283	0.0000
20 percent liquid (80% direct real estate)					
Tracking error with respect to DRE (%)	0.6315	2.1122	0.5794	0.8039	0.0000
Average return (%)	1.9705	2.2860	2.0428	2.1780	2.2065
Average excess returns (%)	-0.2360	0.0795	-0.1637	-0.0285	0.0000
Standard deviation (%)	2.4056	4.1186	2.6226	2.9467	3.0216
Sharpe ratio	0.3823	0.2999	0.3782	0.3825	0.3825
Index value	604.28	797.66	644.99	723.49	742.18
Maximum drawdown	0.2771	0.4482	0.4642	0.4173	0.4219
Change in return with respect to DRE (%)	-10.694	3.6026	-7.4190	-1.2899	0.0000
30 percent liquid (70% direct real estate)					
Tracking error (%)	0.9472	3.1683	0.8690	1.2063	0.0000
Average return (%)	1.8525	2.3257	1.9611	2.1616	2.2065
Average excess returns (%)	-0.3540	0.1192	-0.2454	-0.0449	0.0000
Standard dev. (%)	2.1010	4.9220	2.4503	2.9905	3.0216
Sharpe ratio	0.3816	0.2590	0.3715	0.3714	0.3825
Index value	544.37	814.27	600.56	711.33	742.18
Max drawdown	0.2381	0.4940	0.5153	0.4518	0.4507
Change in return wrt DRE (%)	-16.041	5.4040	-11.124	-2.0345	0.0000
100% liquid (pure replication)					
Tracking error with respect to DRE (%)	3.1574	10.5609	2.8967	4.0173	0.0000
Average return (%)	1.0267	2.6040	1.3884	1.9938	2.2065
Average excess returns (%)	-1.1798	0.3975	-0.8181	-0.2128	0.0000
Standard deviation (%)	0.6237	11.7270	2.1409	4.4406	3.0216
Sharpe ratio	-0.0388	0.1324	0.1577	0.2123	0.3825
Index value	254.44	693.23	355.74	581.39	742.18
Maximum drawdown	0.0000	0.7580	0.7358	0.6301	0.5708
Change in return with respect to DRE (%)	-53.471	18.013	-37.077	-9.6420	0.0000

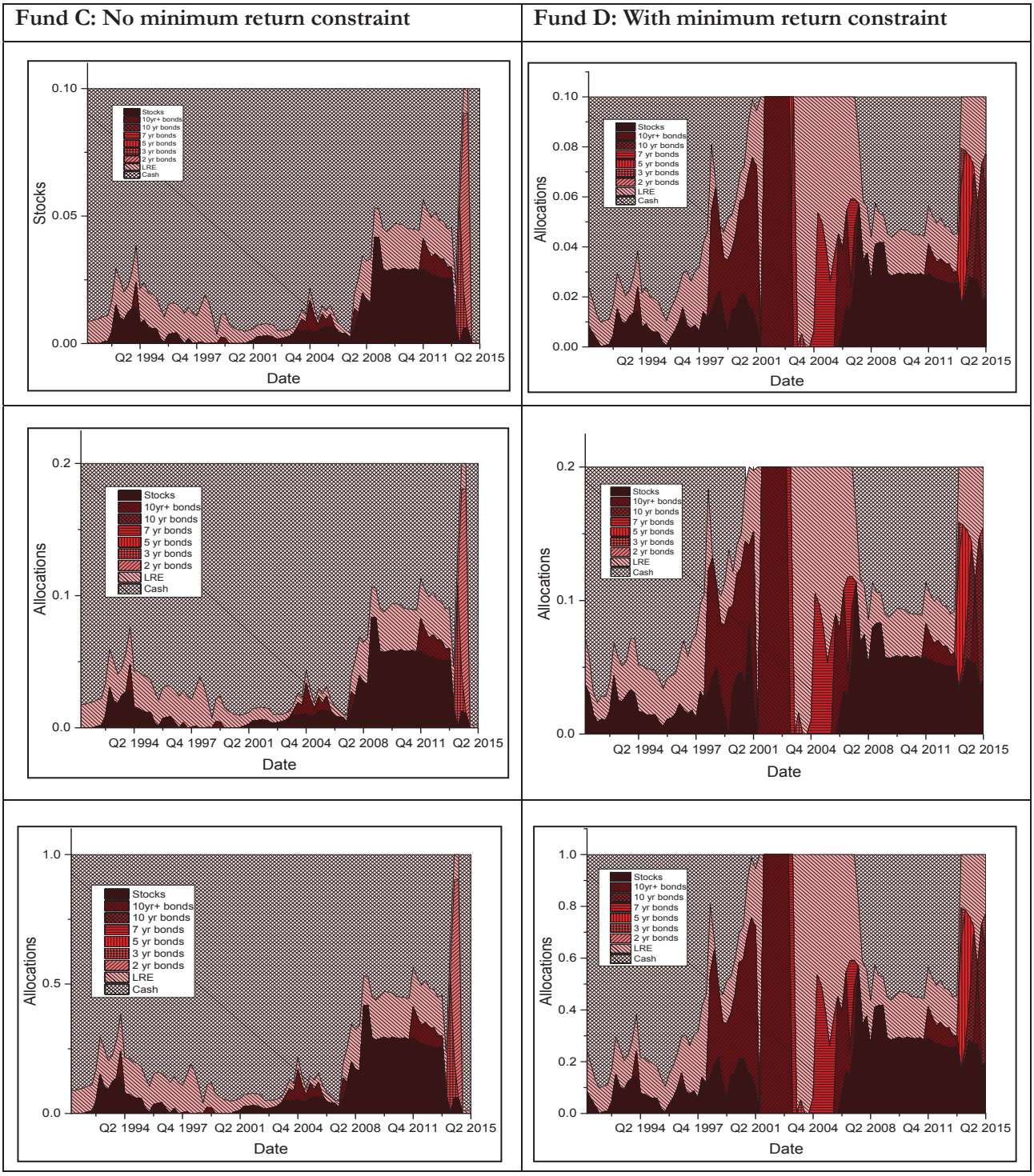
Notes: DRE = IPD All-Property Portfolio; Fund A = A naïve mix of cash and the IPD All Property Portfolio; Fund B = A naïve mix of listed real estate and the IPD All Property Portfolio; Fund C = An optimised blend of the IPD All Property Portfolio and the all selected liquid assets. Optimisation is done without any minimum return constraint; Fund D = An optimised blend of the IPD All Property Portfolio and the all selected liquid assets. Optimisation is done with a constraint that target return should be equal to or greater than the average return on the IPD All Property Portfolio

Table V: Comparison of Static, 4 and 20 Quarter Rolling, and DCC Estimates

10% liquid	**Fund A (DRE-CASH)**	**Fund B (DRE-LRE)**	**Fund B (DRE-ALL)**	**Fund C (DRE-ALL1)**
Dynamic conditional correlation	0.9989	0.8931	0.9941	0.9629
Static correlation	0.9997	0.9555	0.9976	0.9911
4 quarter rolling correlation	0.9999	0.7013	0.9823	0.9010
20 quarter rolling correlation	0.9999	0.8918	0.9966	0.9663
20% liquid				
Dynamic conditional correlation	0.9950	0.7839	0.9772	0.8992
Static correlation	0.9987	0.8691	0.9889	0.9640
4 quarter rolling correlation	0.9996	0.4911	0.9422	0.7356
20 quarter rolling correlation	0.9993	0.7663	0.9843	0.8969
30% liquid				
Dynamic conditional correlation	0.9864	0.6959	0.9504	0.8280
Static correlation	0.9961	0.7839	0.9710	0.9195
4 quarter rolling correlation	0.9987	0.3684	0.8880	0.6098
20 quarter rolling correlation	0.9980	0.6763	0.9600	0.8206
100% Liquid				
Dynamic conditional correlation	-0.1067	0.4558	0.2146	0.3387
Static correlation	-0.1194	0.4956	0.4114	0.4736
4 quarter rolling correlation	-0.2505	0.1328	0.0755	0.0263
20 quarter rolling correlation	-0.1934	0.4369	0.3671	0.3824

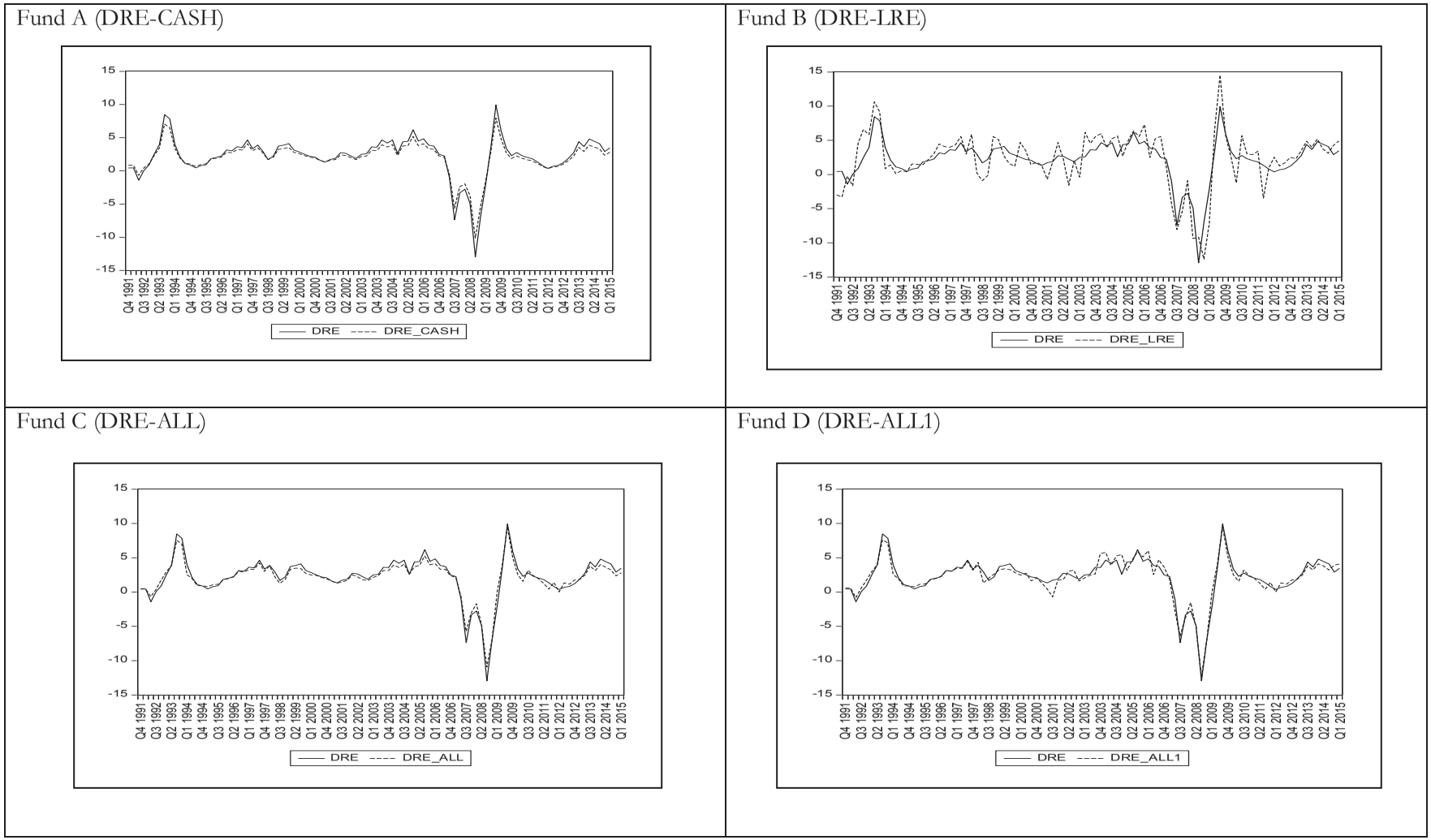
Notes: DRE = IPD All-Property Portfolio; Fund A = A naïve mix of cash and the IPD All Property Portfolio; Fund B = A naïve mix of listed real estate and the IPD All Property Portfolio; Fund C = An optimised blend of the IPD All Property Portfolio and the all selected liquid assets. Optimisation is done without any minimum return constraint; Fund D = An optimised blend of the IPD All Property Portfolio and the all selected liquid assets. Optimisation is done with a constraint that target return should be equal to or greater than the average return on the IPD All Property Portfolio; DCC = Dynamic Conditional Correlation

Figure 1: Allocations within Blended Real Estate Portfolios (Out of Sample)



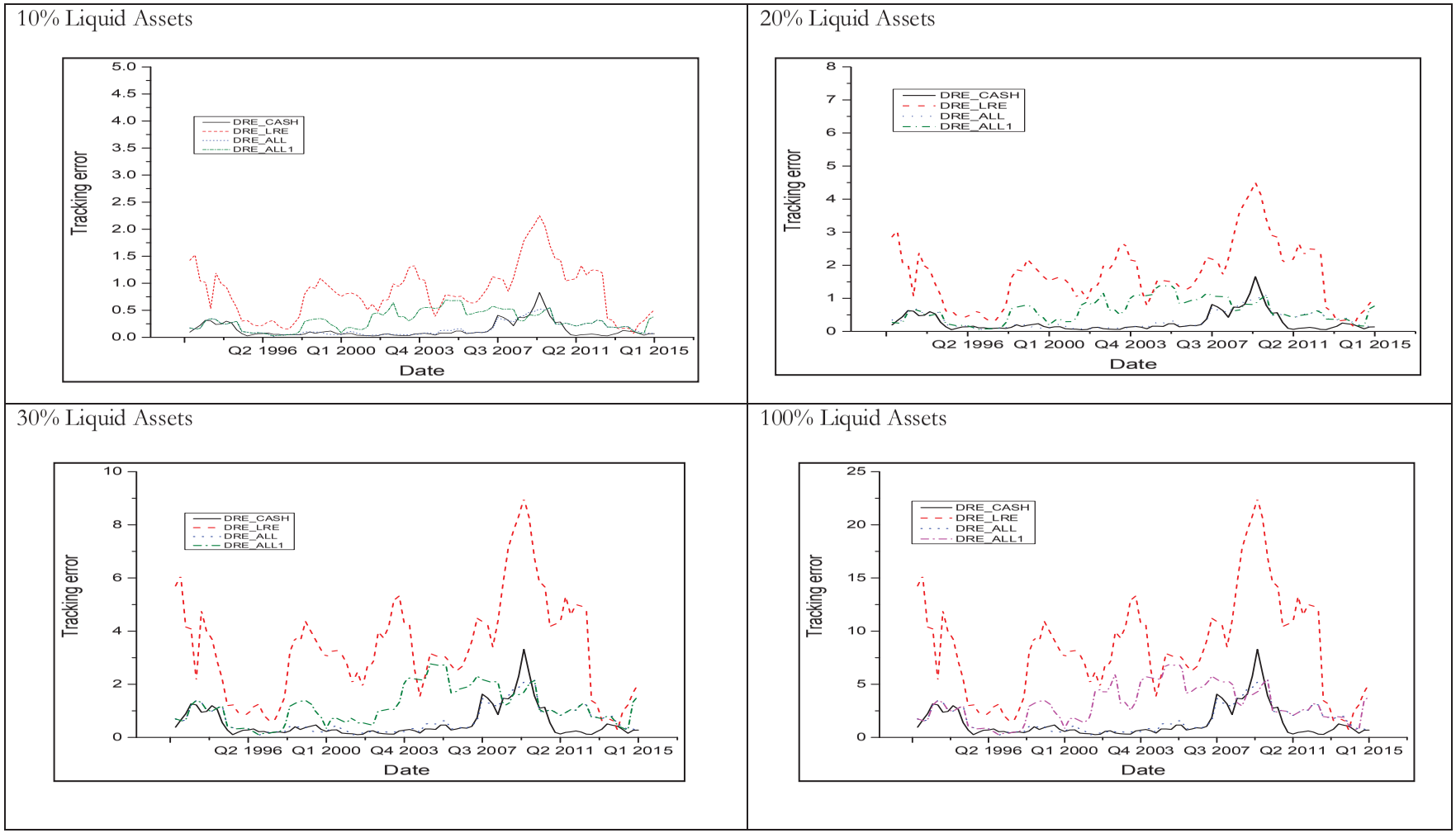
Notes: Bonds 10yr+ = Bonds with maturity greater than 10 years; Bonds 10yr = 10 year bonds; Bonds 7yr = 7 year bonds; Bonds; 5yr = 5 year bonds; Bonds 3yr = 3 year bonds; Bonds 2yr = 2 year bonds; Stocks = General stocks; LRE = Listed real estate; Cash = 3-month T-bills; Fund C = Optimisation is done without any minimum return constraint; Fund D = Optimisation is done with a constraint that target return should be equal to or greater than the average return on the IPD All Property Index.

Figure 2: Out of Sample Returns of Blended Real Estate Portfolios (20% Allocation to Liquid Assets)



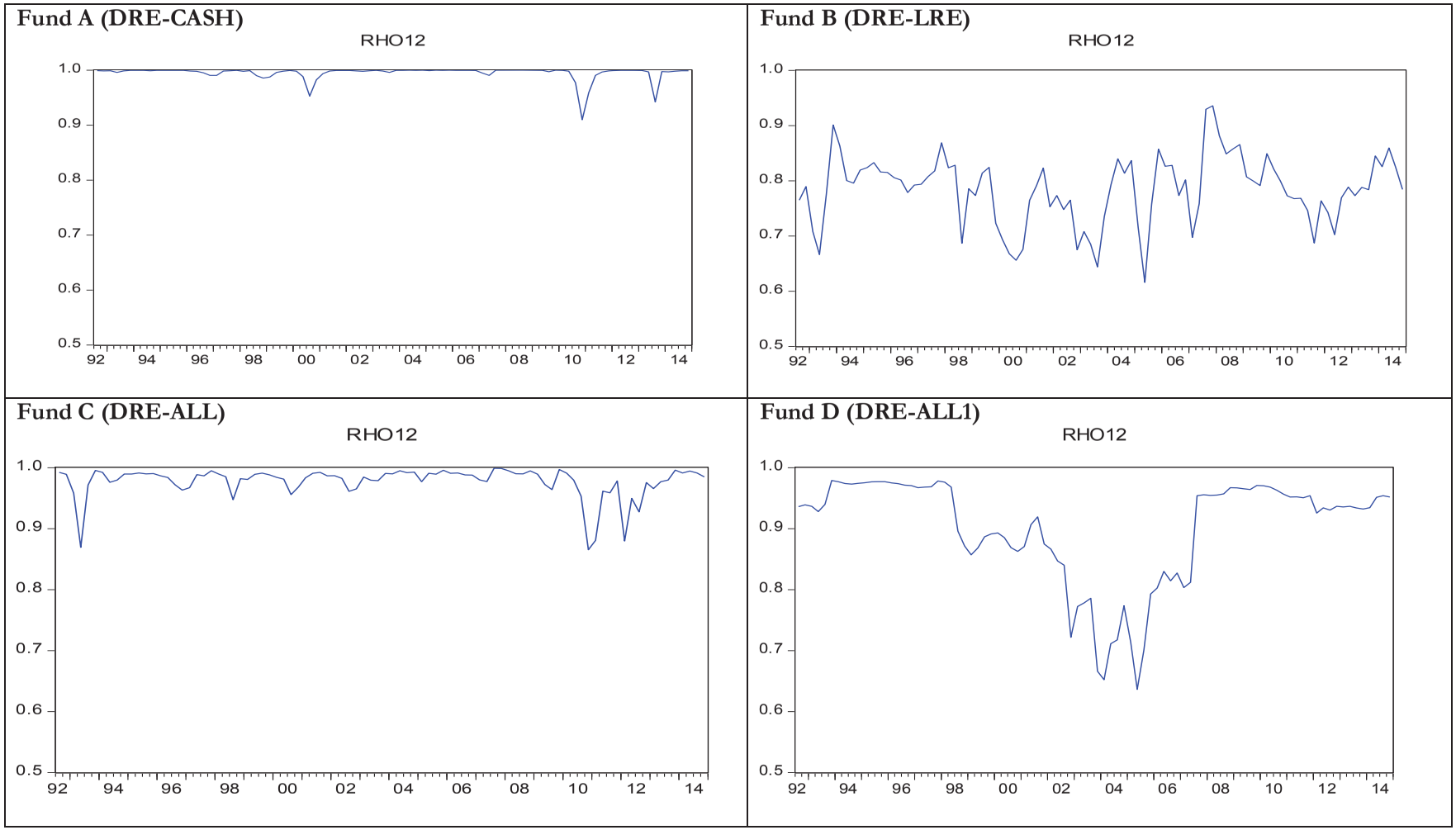
Notes: DRE = IPD All Property Index; Fund A = A naïve mix of cash and the IPD All Property Index; Fund B = A naïve mix of listed real estate and the IPD All Property Index; Fund C = An optimised blend of the IPD All Property Index and selected liquid assets. Fund D = An optimised blend of the IPD All Property Index and selected liquid assets. Optimisation is done with a constraint that target return should be equal to or greater than the average return on the IPD All Property Index.

Figure 3: 20-Quarter (5 year) Rolling Tracking Error



Notes: DRE = IPD All-Property Portfolio; DRE-CASH = A naïve mix of cash and the IPD All Property Portfolio; DRE-LRE = A naïve mix of listed real estate and the IPD All Property Portfolio; DRE-ALL = An optimised blend of the IPD All Property Portfolio and the all selected liquid assets. Optimisation is done without any minimum return constraint; DRE-ALL1 = An optimised blend of the IPD All Property Portfolio and the all selected liquid assets. Optimisation is done with a constraint that target return should be equal to or greater than the average return on the IPD All Property Portfolio

Figure 4: Dynamic Conditional Correlations between Direct Real Estate and Hybrid Real Estate Returns (20% Liquid)



Notes: DRE = IPD All-Property Portfolio; DRE-CASH = A naïve mix of cash and the IPD All Property Portfolio; DRE-LRE = A naïve mix of listed real estate and the IPD All Property Portfolio; DRE-ALL = An optimised blend of the IPD All Property Portfolio and the all selected liquid assets. Optimisation is done without any minimum return constraint; DRE-ALL1 = An optimised blend of the IPD All Property Portfolio and the all selected liquid assets. Optimisation is done with a constraint that target return should be equal to or greater than the average return on the IPD All Property Portfolio; RHO 12: Dynamic Conditional Correlation

Appendix A: Allocation within Balanced UK Unlisted Real Estate Fund Portfolios as at March 2017

Name of Fund	Fund Type	Retail	Offices	Industrials	Others	Cash	Total
AEW UK - Core Property Fund	PAIF	33.40	28.60	23.50	9.60	4.80	100
AEW UK Real Return Fund	PAIF	37.80	4.90	0.00	54.50	2.80	100
Aviva Investors Pensions Limited	MPF	29.90	38.80	15.30	8.80	7.30	100
BlackRock UK Property Fund	PUT	28.00	25.70	23.20	16.90	6.40	100
CBRE UK Property PAIF	PAIF	35.90	23.50	26.40	9.30	4.90	100
COIF Charities Property Fund	PUT	23.10	44.30	30.80	1.60	0.10	100
Fidelity UK Real Estate Fund	EPUT	16.30	39.20	34.50	-	9.90	100
UK Property Fund	PUT	32.30	30.80	31.90	2.60	2.40	100
Hermes Property Unit Trust	EPUT	22.30	38.10	22.70	12.50	4.40	100
Kames Active Value Property Fund	MPT	45.10	25.50	15.50	4.20	9.70	100
Kames Capital UK Active Value Property Unit Trust	PUT	34.40	36.30	13.50	2.80	13.00	100
Keills Property Trust	EPUT	16.90	24.70	11.40	42.80	4.30	100
Legal and General Assurance (Pensions Management) Ltd	MPF	22.80	34.30	16.10	12.70	14.10	100
Lothbury Property Trust	PUT	47.40	25.80	15.30	7.10	4.40	100
Mayfair Capital Property Income Trust for Charities	EPUT	22.40	28.80	32.70	11.10	5.00	100
Mayfair Capital Property Unit Trust	PUT	26.60	42.30	27.70	1.50	2.00	100
Rockspring Hanover Property Unit Trust	PUT	26.80	26.80	45.20	-	1.10	100
Royal London Property Fund	PAIF	33.20	29.40	20.40	11.00	6.00	100
Savills IM UK Income & Growth	PUT	36.90	6.00	40.30	13.40	3.40	100
Schroder UK Real Estate Fund	PAIF	24.50	38.00	19.10	10.00	8.40	100
Standard Life Investments Pooled Pension Property Fund	MPF	39.10	31.60	20.10	0.20	8.90	100
The Charities Property Fund	CIF	28.80	20.80	22.70	23.30	4.50	100
The Local Authorities Property Fund	EPUT	26.30	39.90	25.10	1.00	7.70	100
The M&G UK Property Fund	FCP	35.70	23.10	22.70	10.80	7.90	100
Threadneedle Pensions Ltd	MPF	38.00	25.80	19.30	6.20	10.60	100
Threadneedle Property Unit Trust	PUT	32.80	33.30	24.80	5.10	3.90	100
UBS Triton Property Fund	PNP	34.50	20.00	32.70	12.60	0.20	100

Fund type abbreviations: CIF - Common Investment Fund; EPUT - Exempt Property Unit Trust; FCP - Fonds Commun de Placement; MPF - Managed Pension Fund; PAIF - Property Authorised Investment Fund; PNP - Balanced Property Partnership; PUT - Property Unit Trust.

Source: AREF (2017)